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DISPERSION RELATIONS FOR WEAK INTERACTIONS

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A B S T R A C T:

Dispersion relations for weak interaction processes are obtained in the present paper.

It is shown that in processes which involve not only weakly interacting but strongly interacting particles as well, the dispersion relations are equivalent to the statement that the unknown amplitude functions, which are determined by strong interactions, depend only on momentum transfer between strongly interacting particles.

I. Introduction

One of the main principles of present-day local field theory is the causality principle[1]. It has been recently demonstrated that this principle underlies the deduction of the so-called disperions relations which connect the Hermitian and anti-Hermitian parts and anti-~~Hermitian~~ parts of the process amplitudes. Consequently, an experimental check of these relations may serve to verify localizability of the theory. The dispersion relations are usually deduced for strong interactions. However it would be interesting to ascertain just what the dispersion relations yield in the case of weak interactions.

According to the general theory of dispersion relations[2], the "Hermitian" part of the amplitude D in a special coordinate system is equal to an integral of the "anti-Hermitian" part A, taken over the energy, plus an arbitrary polynomial P_n (The "anti-Hermitian" part of the amplitude can be expressed in terms of the product of the "meson" ("electron") current with the "neutrino" current. Because of smallness of the weak interaction coupling constant, C , only terms involving the first power of the coupling constant should be taken into account. Since the smallness of the product of the aforementioned currents is not lower than the second order the "anti-Hermitian" part will vanish in the approximation under consideration.

In the case of electromagnetic interactions besides processes for which the "anti-Hermitian" part in the given approximation also vanishes (e.g. scattering of electrons on protons) there are some processes (photoproduction, Compton effect) for which the "anti-Hermitian" part is of the same order of magnitude as the "Hermitian". Thus for all weak interactions and for those electromagnetic proseses whose "anti-Hermitian" part of the amplitude is zero, the dispersion relations assume a particularly simple form

$$D(E) = P_n(E) \tag{1}$$

As is well known, the necessity of analyzing the analytical properties of A is the source of the main difficulties which are encountered when one attempts to prove the dispersion relations. Since in the cases under consideration A should be replaced by zero the proof is evident and can be directly obtained from general principles as well as from the usual theory.

In view of the particular simplicity of dispersion relations (1) it would seem to be of special interest to use them to check the localizability of the theory.

2. Dispersion Relations for Weak Interactions

As an example consider the reaction*

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* All conclusions concerning reaction (2) are completely applicable to the reactions $e^- + p \rightarrow n + \nu$.

$$\mu^- + p \rightarrow n + \nu \qquad (2)$$

In this reaction weakly interacting (μ, ν) as well as strongly interacting (p,n) particles are involved. We write the matrix element for process (2) in the form

$$S(p, q, p', q') = (2\pi)^3 \langle \Phi_{p's'} a_\nu(q') S a_\mu^+(q) \Phi_{ps} \rangle \qquad (3)$$

where Φ_{ps} is the state vector of the initial nucleon, a_μ^+ is the μ meson creation operator, and a_ν is the neutrino absorption operator.

Transposing the creation operator in (3) to the position at the extreme left and the annihilation operator to the position at the extreme right we obtain

$$S(p, q, p', q') = \tilde{u}_\nu(q') \int e^{i(q'x - qy)} \langle p's' \Big/ \frac{\delta^2 S}{\delta \bar{\Psi}_\nu(x) \delta \Psi_\mu(y)} S^+ \Big/ ps \rangle u_\mu(q)\, dx\, dy. \qquad (4)$$

Introducing the "meson" and "neutrino" current operators

$$j_\mu(y) = -i \frac{\delta S}{\delta \bar{\Psi}_\mu(y)} S^+; \qquad j_\nu(x) = i \frac{\delta S}{\delta \bar{\Psi}_\nu(x)} S^+ \qquad (5)$$

we get

$$\frac{\delta^2 S}{\delta\bar{\Psi}_\nu(x)\delta\Psi_\mu(y)} S^+ = \begin{cases} -i\dfrac{\delta\bar{j}_\mu(y)}{\delta\bar{\Psi}_\nu(x)} - \bar{j}_\mu(y) j_\nu(x) \\ \\ -i\dfrac{\delta j_\nu(x)}{\delta\Psi_\mu(y)} + j_\nu(x)\bar{j}_\mu(y) \end{cases} \tag{6}$$

However, in virtue of the causality principle[1/]

$$\frac{\delta j_\nu(x)}{\delta\Psi_\mu(y)} = 0 \quad y \gtrsim x; \qquad \frac{\delta\bar{j}_\mu(y)}{\delta\bar{\Psi}_\nu(x)} = 0 \quad x \gtrsim y \tag{7}$$

and therefore taking the first term in the expansion in the weak interaction constant C we get

$$\left\{\frac{\delta^2 S}{\delta\bar{\Psi}_\nu(x)\,\delta\Psi_\mu(y)} S^+\right\}_C = 0 \qquad \text{for } x \neq y \tag{8}$$

This signifies that the given expression is a quasi-local operator and involves $\delta(x-y)$ and possibly its derivatives. The implication is that an equivalent Lagrangian exists which is local with respect to the meson-neutrino interaction and that the unknown matrix element (3) can be obtained by applying perturbation theory to the equivalent Lagrangian. If, as is customary, we assume that derivatives of the meson and neutrino fields do not enter the interaction Lagrangian we get

$$\left\{\frac{\delta^2 S}{\delta\bar{\Psi}_\nu(x)\,\delta\Psi_\mu(y)} S^+\right\}_C = \delta(x-y)\,\Lambda(x). \tag{9}$$

Taking into account (9) we obtain

$$\int e^{i(q'x-qy)} \langle p's' \Big/ \frac{\delta^2 S}{\delta\bar{\psi}_\nu(x)\,\delta\psi_\mu(y)} S^+ \Big/ ps \rangle \, dx\,dy =$$

$$= \int e^{i(q'x-qx)} \langle p's'/\Lambda(x)/ps \rangle \, dx =$$

$$(2\pi)^h \delta(p'+q'-p-q) \langle p's'/\Lambda(0)/ps \rangle$$

(10)

Expression (10) depends on the momenta p and $q'-q$ but not on the momentum $q+q'$. In a coordinate system in which $\vec{p}+\vec{p}\,'=0$ this means that expression (10) is a function of p and is independent on E.

Substituting (10) in (4) and taking into account relativistic invariance requirements we obtain

$$S(p,q,p'q') = \sum_{i,j} (\bar{u}_\nu(q')O^i u_\mu(q))(\bar{u}_n(p')\Omega_{ij} u_p(p^*)F_{ij}((p-p')^2) \qquad (11)$$

where O_i **are basis Dirac matrices,**

Ω_{ij} **are operators consisting of matrices**

γ_μ **and momenta p and p'.**

A consequence of localizability of the meson-neutrino interaction is that the unknown functions ~~determined~~ of a process determined by strong interactions depend only on transfer of momentum to the nucleon. It should be noted that expression (11) was obtained without assuming localizability of strong interactions.

The independence of functions $F_{i\gamma}$ on the momentum $q + q'$ established above can also be deduced from the usual theory by applying the Feynman graph technique. The most general form of diagrams contributing to the process is represented in Fig. 1.

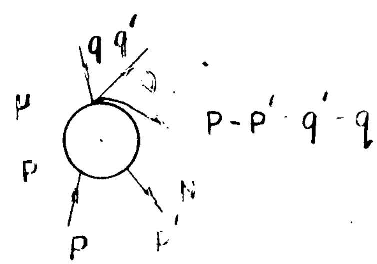

Fig. 1.

Since the meson and neutrino lines converge at a single point in the diagrams (this is a reflection of the localizability condition in the given case), it is apparent that the strong interaction part of the graph depends only on the momenta p and $p - p'$ but not on $q + q'$, i.e., we again obtain expression (11).

It should be particularly emphasized that our main conclusion, namely, that the form factors $F_{i\gamma}$ depend only on momentum transfer to the nucleon, is based on the assumption of localizability of the meson-neutrino interaction but not of localizability of strong interactions.

Thus only conclusions regarding localizability of the meson-neutrino interaction can be deduced from the dispersion relations for process (2).

Indeed, if the meson-neutrino interaction is non-local (i.e., a suitable "form-factor" is introduced between appropriate lines in Fig.1), the functions F_{ij} will markedly depend on the momentum $q + q'$ *. An experimental study of this dependence

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* In this case in a coordinate system in which $\vec{p} + \vec{p}' = 0$ the functions F_{ij} will depend not only on momentum transfer to the nucleon but on the energy of the incident particle as well.

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should directly yield information on localizability of the meson-neutrino interaction.

A determination of the dependence of F_{ij} on momentum transfer to the nucleon should permit one to determine the "meson-neutrino" structure of the nucleon.

Comparison of the diagram in Fig. 1 with diagrams for scattering of electrons on nucleons indicated that the effective dimension of the "meson-neutrino" structure of nucleon is apparently of the same order of magnitude as that of the "electromagnetic" structure. It should also be noted that the results obtained above are completely applicable to ~~decay~~ β decay** and also to such hyperon and K-meson decay

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** In β decay processes the structure functions are practically constant in virtue of small momentum transfer to the nucleon.

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processes in which μ, e and ν particles participate along with strongly interacting particles.*

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* The reaction $\mu \to e + \nu + \nu$ is a special case. Since only weakly interacting particles are involved here, the localizability condition is equivalent to applicability of the first approximation of perturbation theory to the usual Lagrangian.

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3. K-Meson Decay

In this section we shall apply the results obtained above to K-meson decay

$$K \longrightarrow \pi + e + \tag{12}$$

Assuming the K-mesons to be pseudoscalar and applying the results obtained in section 1 we arrive at the following expression for the decay amplitude

$$\begin{aligned} S(p, p', q, q') = {} & G_1(|p-p'|^2)(\bar{u}_e(q)\, u_\nu(q')) + \\ & + \frac{G_2(|p-p'|^2)}{M}(\bar{u}_e(q)\, \gamma p\, u_\nu(q')) + \\ & + \frac{G_3(|p-p'|^2)}{M^2}(\bar{u}_e(q)\, \sigma_{\mu\nu} p_\mu p'_\nu\, u_\nu(q')) \end{aligned} \tag{13}$$

where p and p' are respectively the K- and π -meson momenta, q and q' are the electron and neutrino momenta and M is the K-meson mass.

As in the foregoing it is assumed that the electron-neutrino interaction is local and does not contain derivatives. A consequence of this is that in the c.m.s. the unknown functions G which are determined by strong interactions, depend on the π meson energy. It is the a verification of this fact that should permit one to check the localizability of the electron-neutrino interaction. This can be done by studying the angular and energy distribution of decay products for fixed values of the π meson energy.

The following expression for the angulardistribution can readily be obtained by using formula (13)[3]

$$W(E_{p'},\theta) = \frac{1}{2(2\pi)^3}\Big\{|G_1|^2(1+x\cos\theta)^2 +$$

$$+|G_2|^2 x^2\sin^2\theta + |G_3|^2\frac{\vec{p}'^2}{M^2}(x+\cos\theta)^2 -$$

$$2\,\mathrm{Re}\Big[mG_1G_3^*\frac{|\vec{p}'|}{M}(x+\cos\theta)(1+x\cos\theta)\Big]\Big\}\times$$

$$\times\frac{(M-E_{p'})^2(1-x^2)^2}{(1+x\cos\theta)^4}|\vec{p}'|E_{p'}\,dE_{p'}\sin\theta\,d\theta \qquad (14)$$

where $\vec{p}'$ and   are respectively the π meson momentum and energy; θ is the angle between the π meson and electron momenta and $x = \frac{|\vec{p}'|}{M - E_{p'}}$. Expression(14) was obtained on the assumption that the electron energy is large compared with its rest mass. The energy distribution can be written in the form

$$W(E_{p'}, E_q) = \frac{1}{(2\text{[illegible]})}\{|G_1|^2(1-x^2) + |G_2|^2(x-1+2y)(x+1-2y) +$$

$$+ |G_3|^2 \frac{(M-E_{p'})^2}{M^2}(1-x^2)(1-2y)^2 - 2\,\mathrm{Im}\,G_1 G_3^* \frac{E_{p'}}{M}(1-x^2)(1-2y)\}$$

$$(M-E_{p'})E_{p'}\,dE_{p'}\,dE_q \qquad (15)$$

where $\vec{q}$ and E_q are the electron momentum and energy and

$$y = \frac{|\vec{q}|}{M - E_{p'}}$$

Comparison of formulae (14) and (15) with experimental data on the angular and energy distributions may be used to check the localizability of the electron-neutrino interaction. It should be mentioned that since the maximum decay electron energy $E_q \sim 200$ MeV, process (12) should permit one to investigate distances of the order of 10^{-13} cm. Agreement between the experimental data and formulae (15) and (16) therefore may merely indicate that the fundamental length, if it exists, does not exceed 10^{-13} cm.

4. Electromagnetic Interactions

It was mentioned in the introduction that in a number of processes involving electromagnetic interaction the "anti-Hermitian" part of the amplitude is of the same order of magnitude as the "Hermitian". The dispersion relations for these cases were derived in a number of papers[4-7] and they will not be discussed here.

A typical process for which the "anti-Hermitian" part of the amplitude is of a higher order of smallness than the "Hermitian" is scattering of electrons on protons.

At present this process is being studied with the purpose of determining the electromagnetic structure of the nucleon[8,9]. It is quite evident that the results obtained in section 1 are completely appliable in this case.

Summing up, it may be stated that for the processes considered above the dispersion relations are equivalent to the statemtnt that the unknown amplitude functions, which are determined by strong interactions, depend only on momentum transfer between strongly interacting particles.

Of course, this property can be actually applied only if an analysis of radiative corrections is carried out since one must know the limits of the region in which these corrections are insignificant.

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